Beyond Morlet - Wavelet Methods for Analyzing Non-stationary Data

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Morlet wavelet analysis

Morlet wavelet: $\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$ where $\psi(t) = \frac{1}{\sqrt{2\pi}}e^{-\jmath\omega_0 t}e^{-t^2/2}$

with $\omega_0 = \pi\sqrt{\frac{2}{\ln 2}}$ → Filtering

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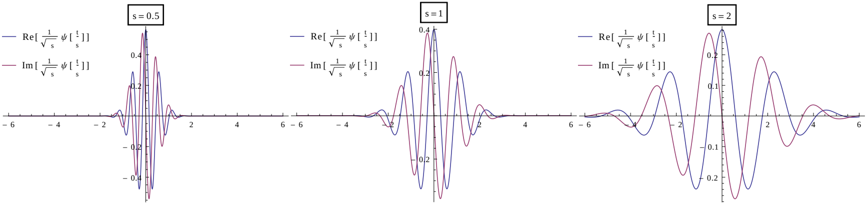

Time domain

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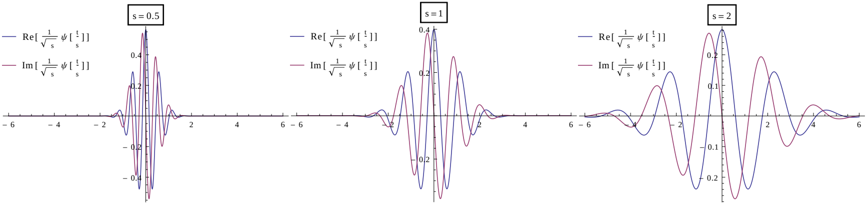
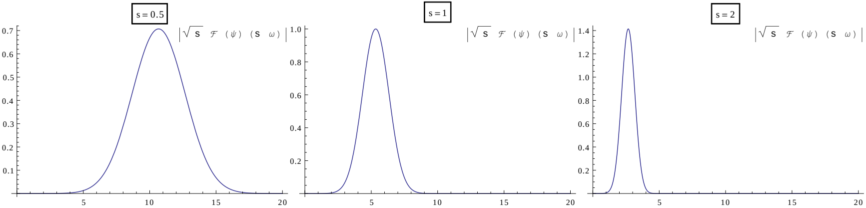


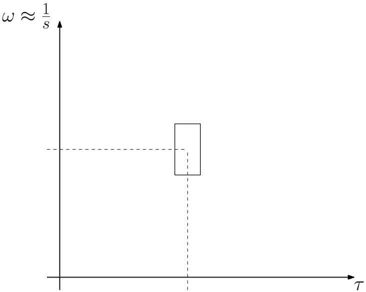
Time domain

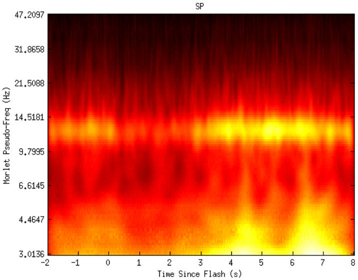


Frequency domain

Time-frequency plane

Goal: find active “instantaneous frequencies” at a given instant τ





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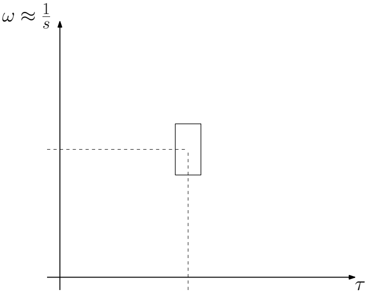

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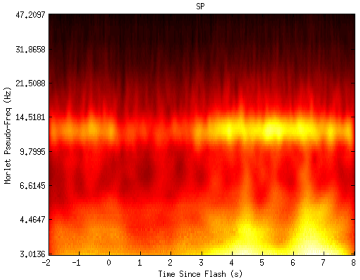
(i.e we don’t have access to the true instantaneous frequencies)



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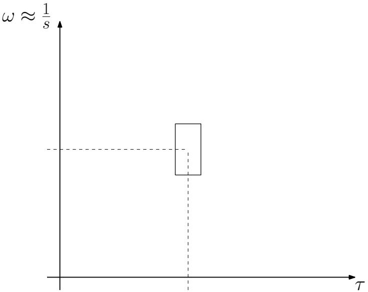

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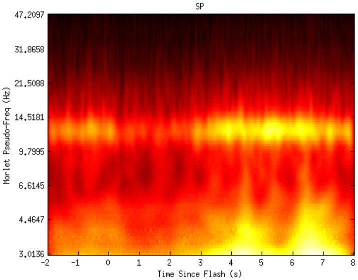
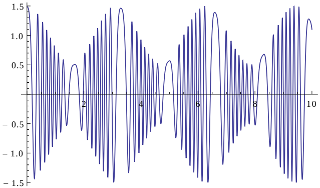


Can we access the true instantaneous information?
→ Empirical Wavelet Transform

Empirical Wavelet Transform: concept 1/2

Useful tool: Hilbert transform applied to AM-FM signals

If $f(t) = F(t)\cos(\varphi(t)) \rightarrow f_a(t) = f(t) + \jmath\mathcal{H}(f(t)) = F(t)e^{\jmath\varphi(t)}$



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- instantaneous amplitude: $F(t) = |f_a(t)|$
- instantaneous frequency: $\omega(t) = \frac{d}{dt}\angle f_a(t)$

TF representation is given by the amplitude $F(t)$ at position $(t, \omega(t))$

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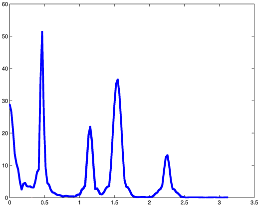
Generalization

Decompose the input signal as a superposition of AM-FM waves

$$f(t) = \sum_{k=0}^{N} f_k(t) \qquad \text{with} \qquad f_k(t) = F_k(t)\cos(\varphi_k(t))$$

then retrieve all $F_k(t)$ and $\omega_k(t)$ by Hilbert transform on each $f_k(t)$ and aggregate the information on the TF plane

Empirical Wavelet Transform: concept 2/2



How to extract AM-FM components?

AM-FM wave $\Leftrightarrow$ "mode in the frequency domain"

Assumption all AM-FM waves are "sufficiently" distinct

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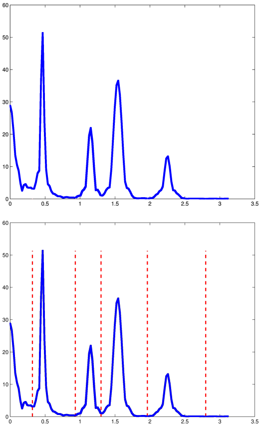

Empirical wavelets transform

- Detect AM-FM supports

Empirical Wavelet Transform: concept 2/2

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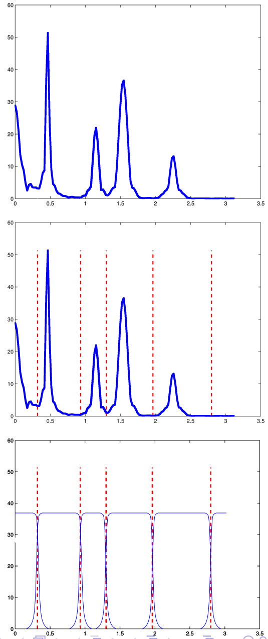
- Detect AM-FM supports
- Build wavelet filters based on detected supports



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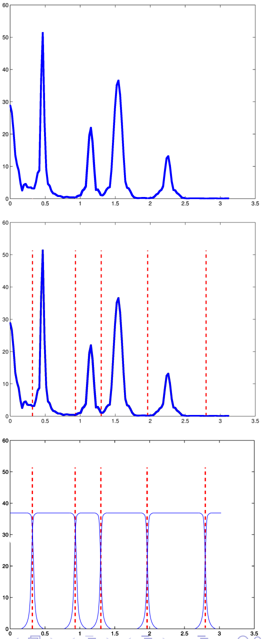

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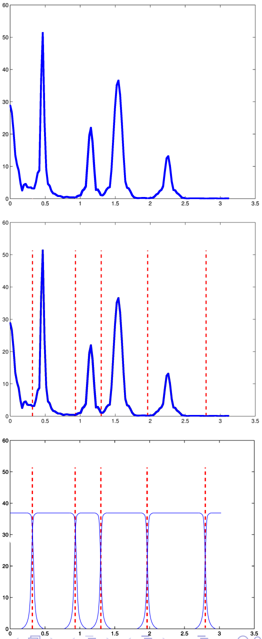
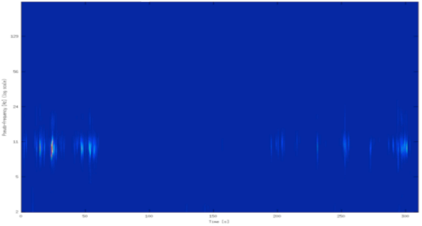
- Use the Hilbert transform and build the TF representation

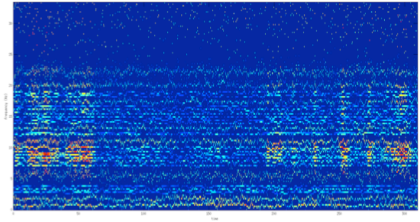


Example on EEG

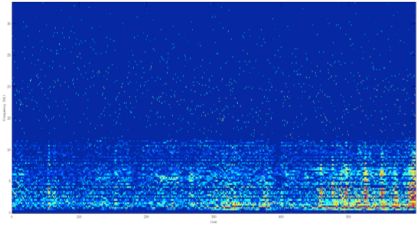
CWT Globus Pallidus



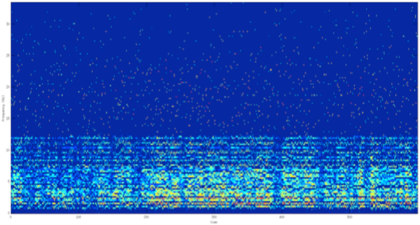
EWT Globus Pallidus



EWT Thalamus



EWT from ECoG Motor Cortex



Conclusion

- EWT provides a more accurate instantaneous Time-Frequency information
- Lot of potential for brain signal analysis (reveal "hidden" patterns, frequency coupling,...)
- Analysis of frequency patterns