Empirical wavelets based texture classification/segmentation

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Textures

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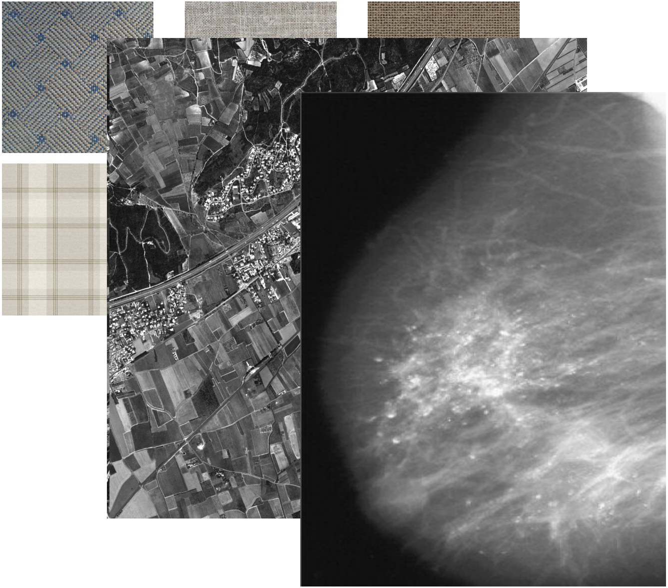

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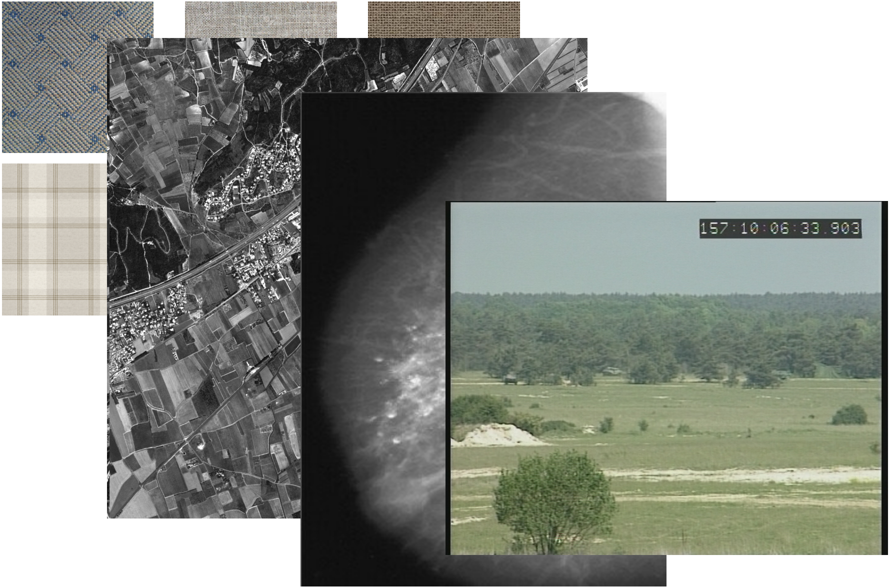

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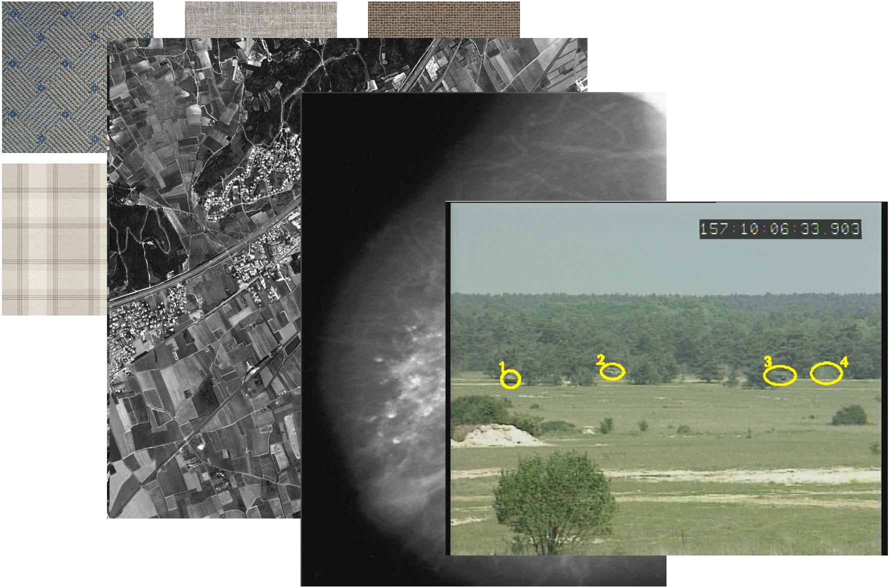

Outline

1. (Classic/)Empirical wavelets
2. Generalities about texture characterization
3. Unsupervised texture segmentation
4. Supervised texture classification

Time-Frequency (TF) analysis (1/2)

Gabor-Heisenberg incertitude principle limited TF:

- Short-time Fourier transform:
  $\mathcal{F}_f^w(m,n) = \int f(s)w(s-nt_0)e^{-\imath m\omega_0 s}ds.$
- Wavelet transform:
  $\mathcal{WT}_f(m,n) = a_0^{-m/2}\int f(t)\psi(a_0^{-m}t-nb_0)dt.$

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How to go beyond this limitation? $\Rightarrow$ Hilbert-Huang transform[1]:

Step 1 Empirical Mode Decomposition (EMD): decompose f as

$$\{f_k\}_{k=0}^N \qquad \text{s.t} \qquad f(t) = \sum_{k=0}^{N} f_k(t)$$

where $f_k(t) = F_k(t)\cos(\varphi_k(t)) \quad \text{s.t} \quad F_k(t), \varphi_k'(t) > 0 \ \forall t.$

Main assumption: F_k and φ_k' vary much slower than φ_k.

[1] The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, Proc. Royal Society London A., vol.454, pp.903–995,1998

Time-Frequency (TF) analysis (2/2)

Step 2 Hilbert Transform (HT):

$$\mathcal{H}_{f_k}(t) = \frac{1}{\pi} p.v. \int_{-\infty}^{+\infty} \frac{f_k(\tau)}{t-\tau} d\tau$$

Property: if $f_k(t) = F_k(t)\cos(\varphi_k(t))$ then

$$f_k^*(t) = f_k(t) + \imath \mathcal{H}_{f_k}(t) = F_k(t) e^{\imath \varphi_k(t)}$$

$\Rightarrow$ easy to extract $F_k(t)$ and the instantaneous frequency $\frac{d\varphi_k}{dt}(t)$.

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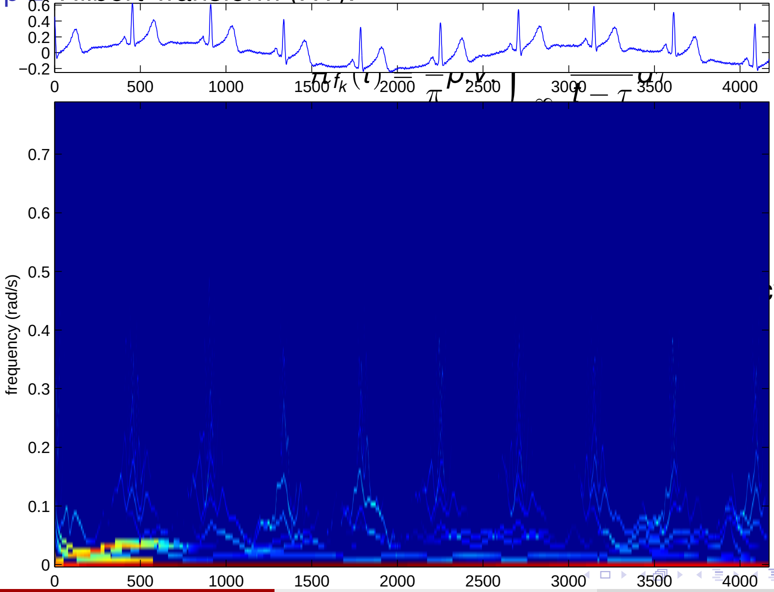

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- EMD is purely algorithmic
- Lacks of mathematical foundation

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Observation: behaves like a data-driven filter bank

Empirical wavelet transform (EWT): Concept

Idea: Build an adaptive (i.e data-driven) wavelet filter bank to replace the EMD.

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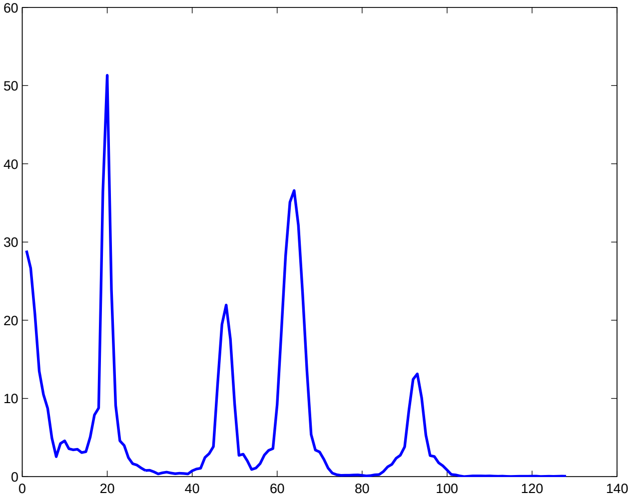
How? AM-FM components ⇔ “compactly supported” spectral modes.



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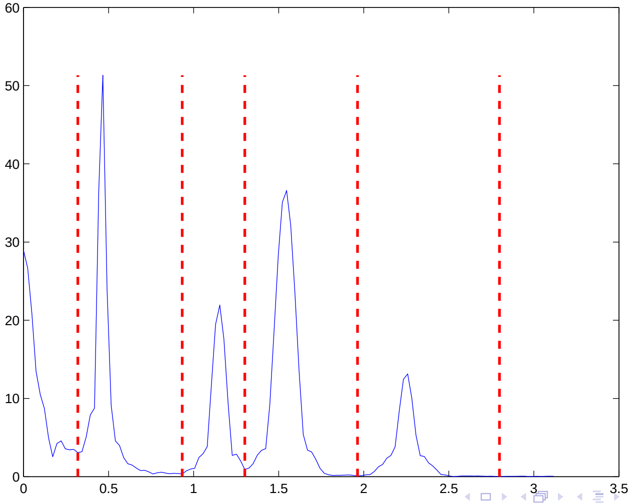

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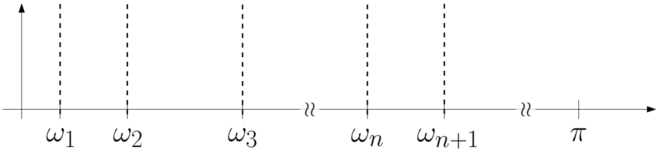
⇒ define set of boundaries $\{\omega_n\}$



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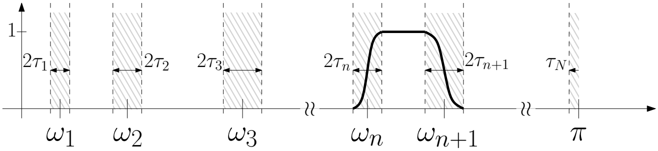
How? AM-FM components $\Leftrightarrow$ "compactly supported" spectral modes.
$\Rightarrow$ segment Fourier spectrum to find the filter supports
$\Rightarrow$ define set of boundaries $\{\omega_n\}$
$\Rightarrow$ define transition areas and then wavelet filters



$$\hat{\phi}_n(\omega) = \begin{cases} 1 & \text{if } |\omega| \leqslant (1-\gamma)\omega_n \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_n}(|\omega| - (1-\gamma)\omega_n)\right)\right] & \text{if } (1-\gamma)\omega_n \leqslant |\omega| \leqslant (1+\gamma)\omega_n \\ 0 & \text{otherwise} \end{cases}$$

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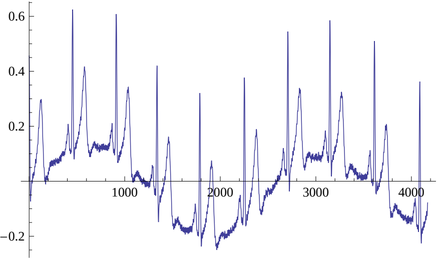
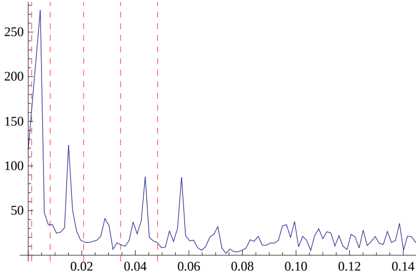
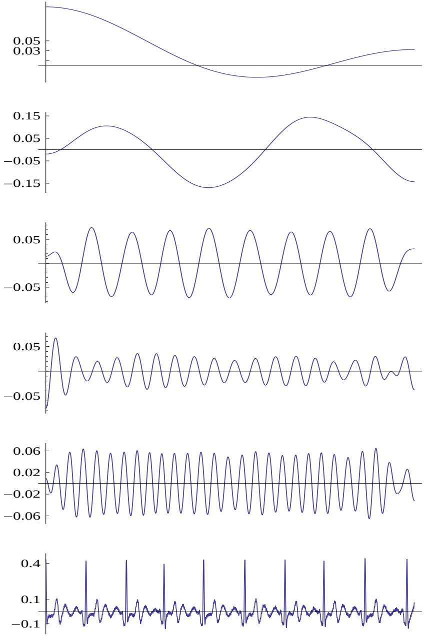
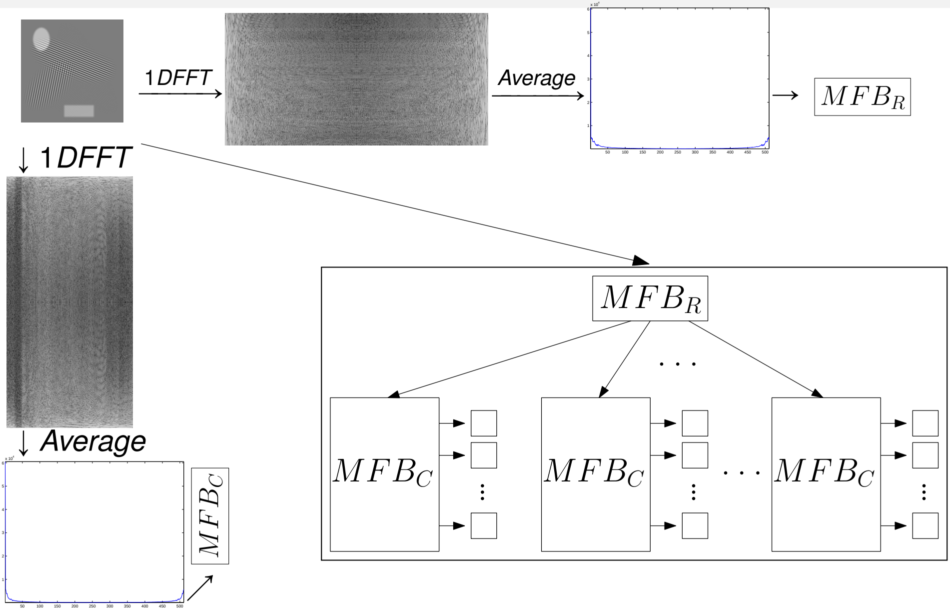
Experiment: ECG



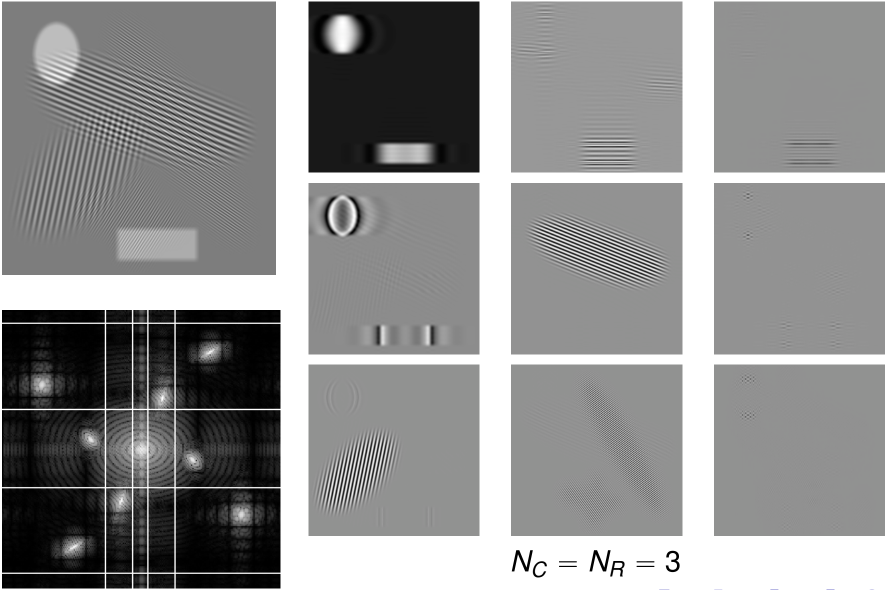




2D Tensor product extension



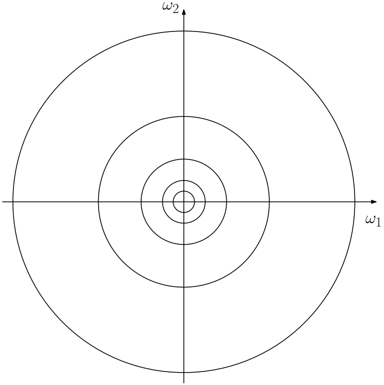
2D Tensor EWT - Example



$N_C = N_R = 3$

2D Empirical Littlewood-Paley Transform (1/2)

Wavelets defined on concentric dyadic annuli in the Fourier plane → radial profile ⇔ 1D dyadic wavelet



2D Empirical Littlewood-Paley Transform (1/2)

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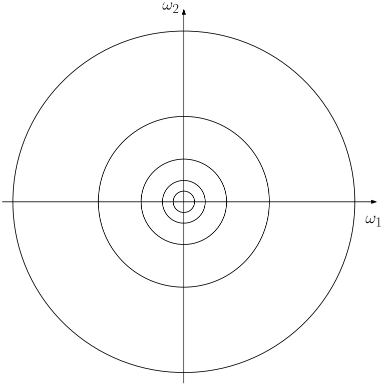
Empirical extension: detect annuli positions



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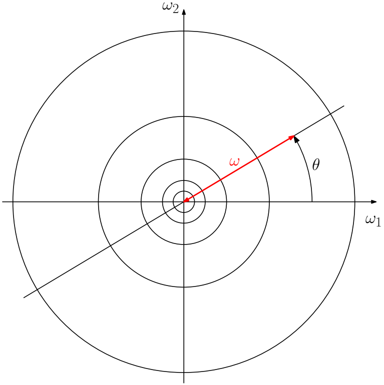
Detect the boundaries over a radial line



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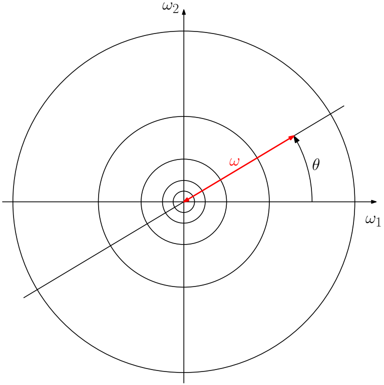
Detect the boundaries over a radial line $\rightarrow$ average spectrum for all θ



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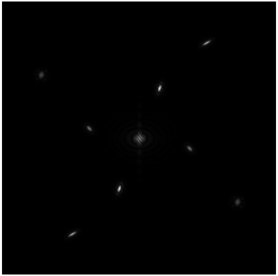
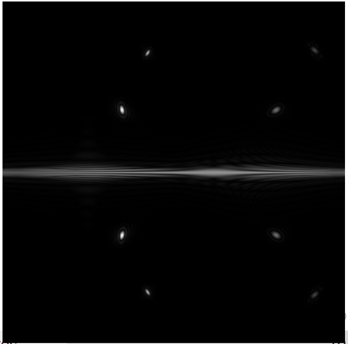
Empirical extension: detect annuli positions

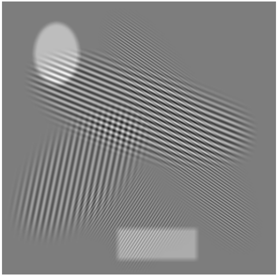
Detect the boundaries over a radial line $\rightarrow$ average spectrum for all θ

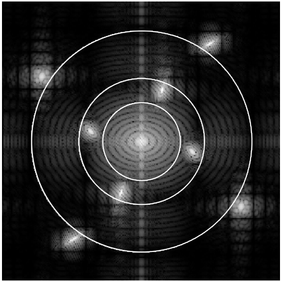
Useful tool: Pseudo-Polar Fourier Transform (PPFT)

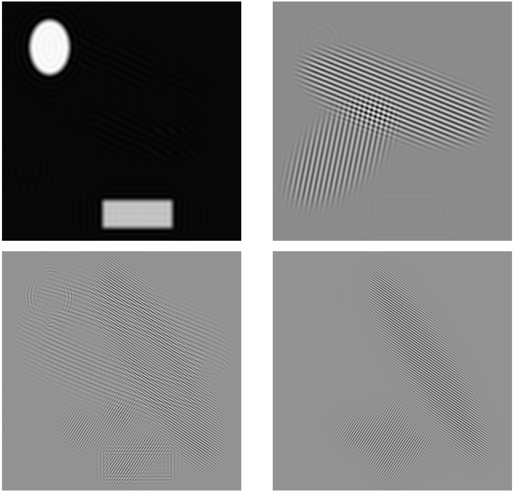




2D Empirical Littlewood-Paley Transform - Example







$N = 4$

Empirical “Curvelet” transforms

Idea: fix scales and angular positions empirically.

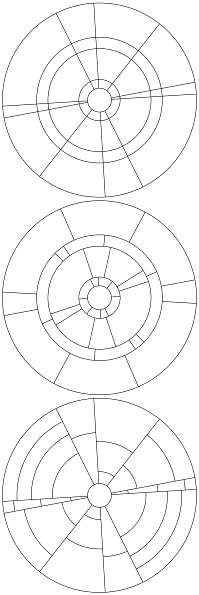
$$\mathcal{F}_2(\psi_{nm})(\omega,\theta) = W_n(\omega)V_m(\theta)$$

Different options:

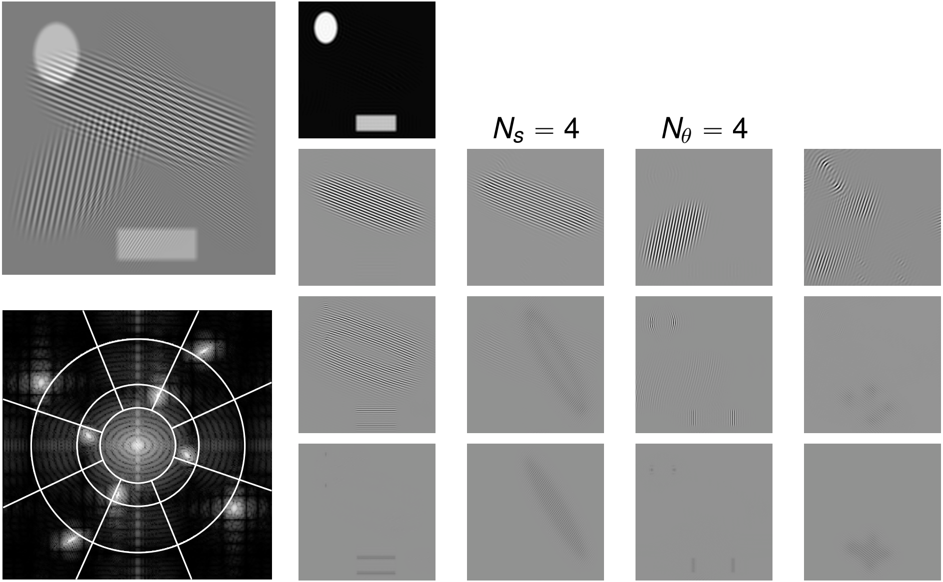
I independent detections:
$\Omega_\omega = \{\omega^n\}_{n=0,\dots N_s}$, $\Omega_\theta = \{\theta^m\}_{m=1,\dots,N_\theta}$

II scales first and then angles per scale:
$\Omega_\omega = \{\omega^n\}_{n=0,\dots N_s}$, $\Omega_\theta^\omega = \{\theta^{n,m}\}_{m=1,\dots,N_\theta}$

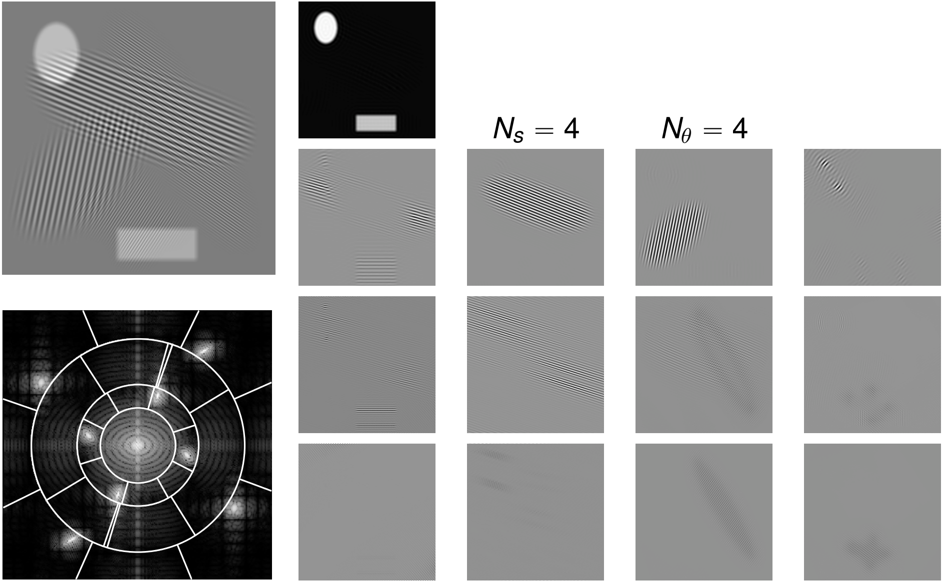
III angles first and then scales per angular sector:
$\Omega_\omega^\theta = \{\omega^{n,m}\}_{n=0,\dots N_s}$, $\Omega_\theta = \{\theta^m\}_{m=1,\dots,N_\theta}$



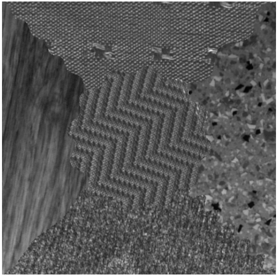
Empirical Curvelet Transform I - Examples



Empirical Curvelet Transform II - Examples



Generalities about texture characterization



Pixel intensities does not permit to distinguish textures!

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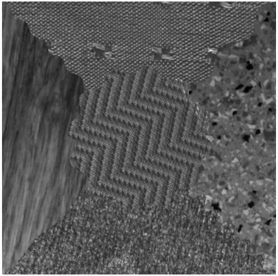

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Important features:

- scale
- periodicity
- orientation

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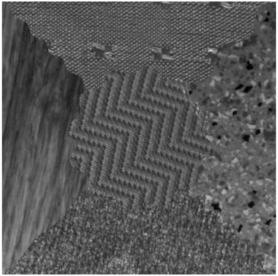

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Important features:

- scale
- periodicity
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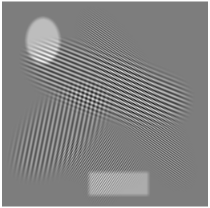
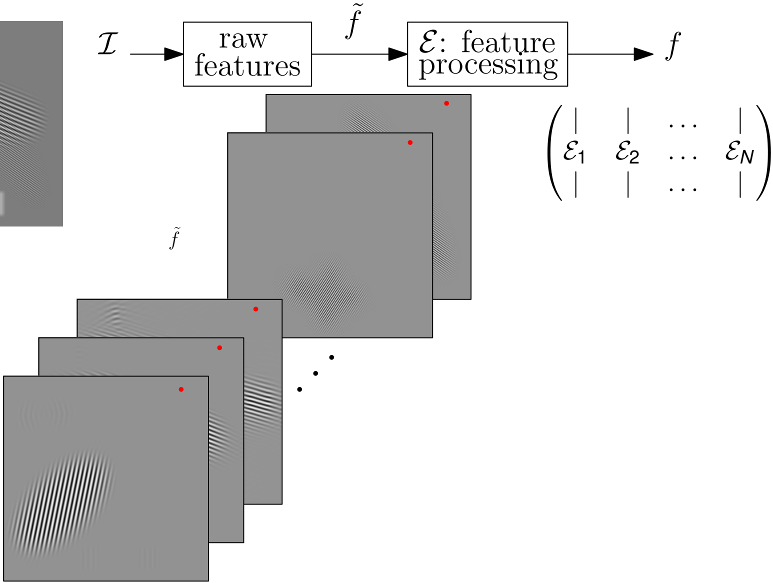
How to extract such features?

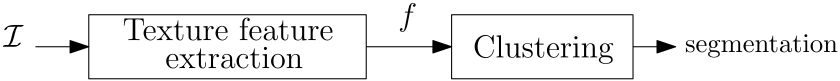
- Gray-Level Co-Occurrence Matrix (GLCM)
- Local Binary Patterns (LBP)
- Stochastic Processes (Markov Random Fields)
- Entropy/Energy of Wavelet coefficients (Gabor, curvelets, . . .)

Wavelet based texture features





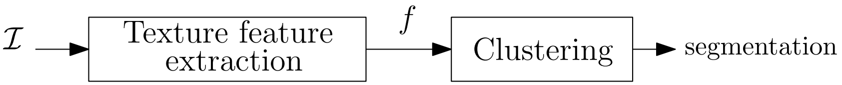
Unsupervised segmentation



Clustering methods (#classes is a parameter):

- k-means
- Nyström algorithm (spectral clustering)
- Variational models (Mumford-Shah, Chan-Vese,...)
- ...

Unsupervised segmentation



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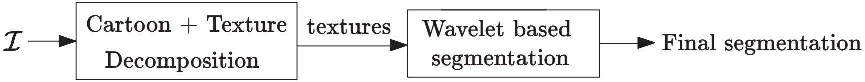
Variant:



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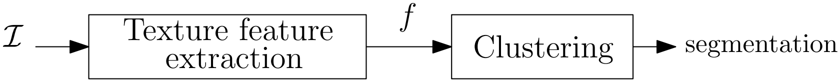
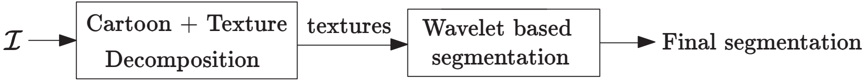
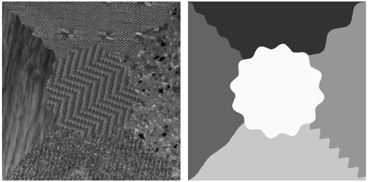


Image Cartoon Texture



Experiments - Setup

- 4 datasets (Brodatz, ALOT, UIUC, Outex), i.e hundreds of images per datasets



- 2 clustering methods (k-means and Nyström) with 4 different metrics
- benchmark metric: average of 6 classic segmentation metrics
- type of feature processing (energy, entropy, LBP) with its kernel size
- 64 different types of wavelets (continuous, discrete, undecimated, empirical)

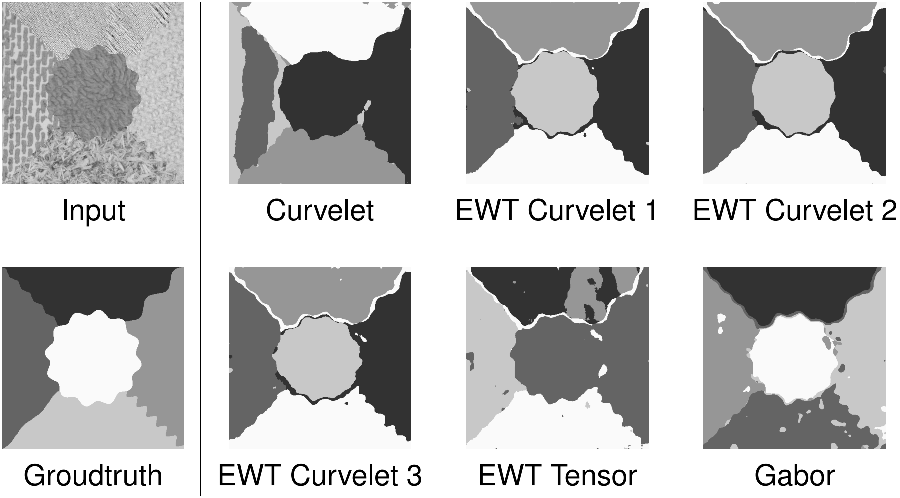
Experiments - Results (1/2)

Best options are:

- k-means with “cityblock” metric
- energy processing with a window size in the range 19-25 (depending on the dataset)
- empirical wavelets:

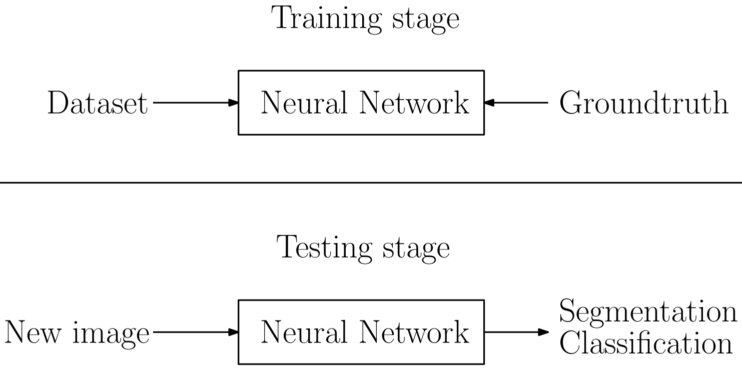
Wavelet	**Outex**	**Brodatz**	**ALOT**	**UIUC**
Curvelet	85.02(4.93)	80.11(7.83)	79.74(8.43)	78.58(7.18)
EWTC1	**87.24(7.94)**	81.04(9.92)	**81.51(9.94)**	**79.32(9.81)**
EWTC2	86.98(8.15)	81.09(9.88)	81.30(9.54)	78.64(9.82)
EWTC3	83.66(11.31)	76.63(14.31)	74.97(13.41)	74.36(15.81)
EWTLP	61.55(11.19)	65.00(13.64)	73.01(12.41)	55.71(13.00)
EWT2DT	82.60(7.50)	**84.01(10.20)**	81.26(10.11)	73.75(11.400
Gabor	81.16(7.49)	82.23(10.58)	80.30(10.53)	73.69(9.80)
Meyer_2	72.68(7.08)	75.24(10.04)	76.56(10.13)	61.34(9.16)
Meyer_3	75.60(6.42)	81.33(8.73)	79.50(9.01)	71.53(9.29)
Meyer_4	75.97(7.17)	80.97(8.86)	80.48(8.37)	76.01(8.29)

Experiments - Results (2/2)



Supervised segmentation - Generalities

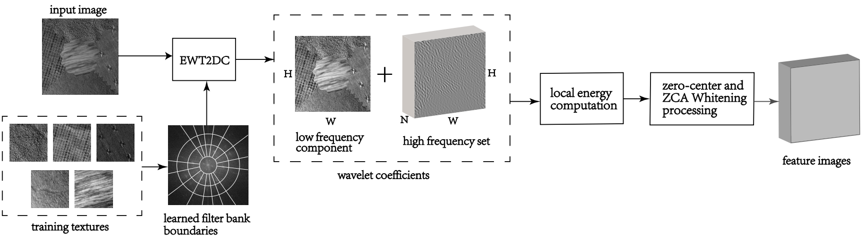
Goal: train the algorithm on a dataset of textures and then test on images containing several textures.



Supervised segmentation - Configurations

Issue: each image in the training set will provide one set of empirical wavelet filters → we need a single set of empirical wavelets per training set → learning the EW filters!

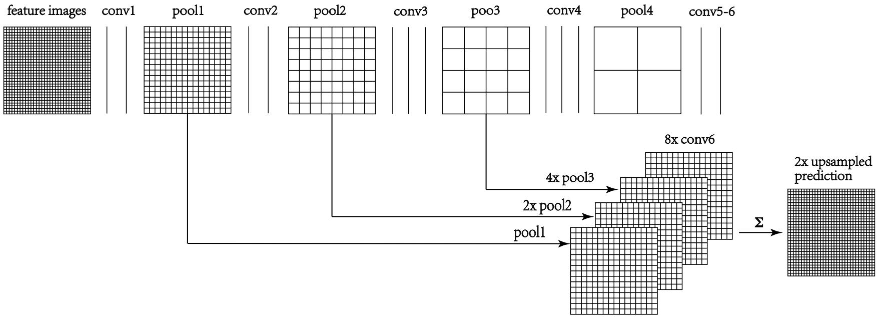
The same set of EW filter is then used in the testing stage.



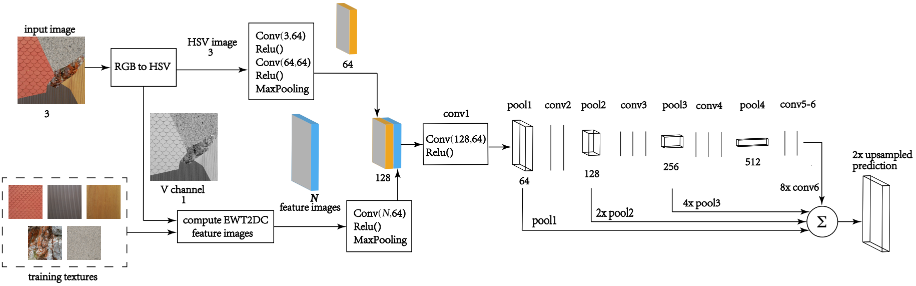
Supervised segmentation - Network architectures

- Fully Convolutional Network for Texture (FCNT)
- U-Net
- Siamese-Net
- Deep Visual Attention model (DA)
- Pyramid Scene Parsing Network (PSP-Net)

Supervised segmentation - Network architectures

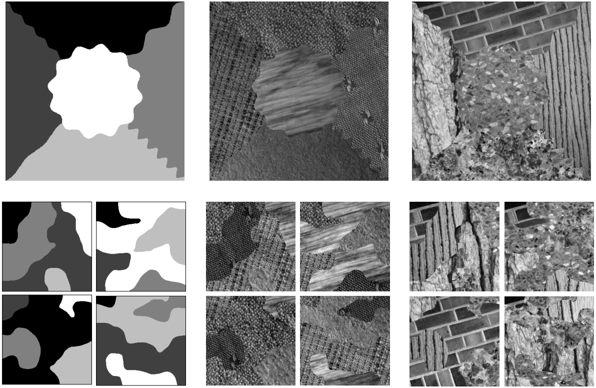


Supervised segmentation - Color case



Grayscale experiments - Training/test sets

Creation of thousand of training/test images with random groundtruths:

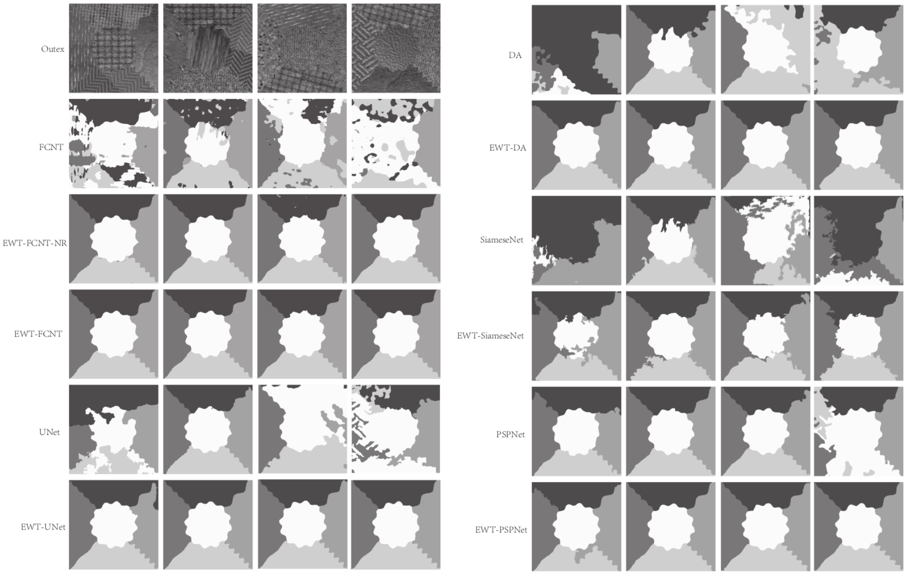


→ Avoid learning the shape of a unique groundtruth

Grayscale experiments - Results (1/2)

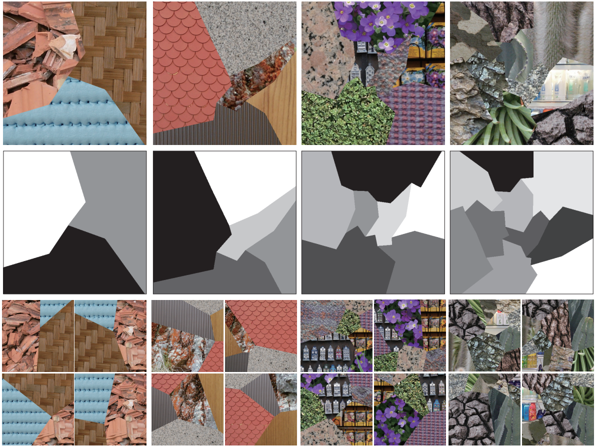
Method	Refinement	NVOI	SSC	SDHD	BGM	VD	BCE	Average	StD
				Outex					
FCNT	no	65.38	62.84	72.91	72.66	77.96	60.09	68.64	11.70
EWT-FCNT	no	96.06	98.00	98.98	98.98	98.98	97.70	98.12	1.22
EWT-FCNT	yes	96.52	98.27	99.13	99.13	99.13	98.02	98.36	0.74
U-Net	yes	72.83	64.25	71.40	70.85	80.47	61.89	70.28	14.86
EWT-U-Net	yes	97.51	98.79	99.39	99.39	99.39	98.54	**98.83**	**0.73**
DA	yes	70.70	60.65	67.77	67.27	78.75	58.58	67.28	16.06
EWT-DA	yes	95.84	97.82	98.89	98.89	98.89	97.45	97.97	0.89
Siamese-Net	yes	67.46	56.77	64.36	63.60	76.46	53.80	63.74	12.89
EWT-Siamese-Net	yes	86.66	88.47	93.09	93.03	93.87	86.32	90.24	6.92
PSP-Net	yes	86.00	84.22	88.00	87.83	91.64	82.63	86.72	14.10
EWT-PSP-Net	yes	96.05	97.93	98.94	98.94	98.94	97.53	98.06	1.04
				UIUC					
FCNT	no	85.81	89.71	94.39	94.39	94.39	87.87	91.09	4.72
EWT-FCNT	no	93.32	95.88	97.85	97.85	97.85	95.05	96.30	2.65
EWT-FCNT	yes	94.66	96.89	98.40	98.40	98.40	96.25	97.17	1.79
U-Net	yes	88.73	90.62	94.72	94.63	94.97	88.64	92.05	5.73
EWT-U-Net	yes	95.87	97.50	98.71	98.71	98.71	96.84	**97.72**	2.07
DA	yes	90.23	92.04	95.59	95.59	95.74	90.47	93.28	4.70
EWT-DA	yes	93.09	95.19	97.43	97.43	97.44	94.16	95.79	3.67
Siamese-Net	yes	75.05	70.37	79.17	78.80	83.85	67.98	75.87	11.47
EWT-Siamese-Net	yes	78.54	78.02	85.54	85.06	87.89	75.00	81.67	10.99
PSP-Net	yes	91.73	94.00	96.76	96.76	96.86	92.65	94.79	3.53
EWT-PSP-Net	yes	94.57	96.78	98.34	98.34	98.34	96.07	97.07	**1.64**

Grayscale experiments - Results (2/2)



Color experiments - Training/test sets

Use of the Prague dataset + creation of new samples:

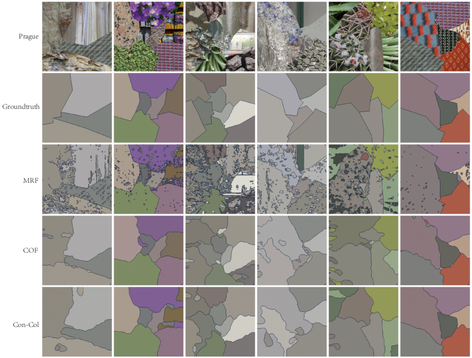


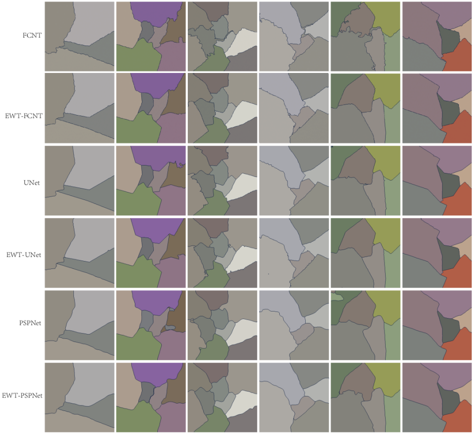
+ use of the metrics defined on the Prague dataset.

Color experiments - Results (1/2)

Method	MRF	COF	Con-Col	FCNT-NR	FCNT	EWT-FCNT-NR	EWT-FCNT	U-Net	EWT-U-Net
↑ CS	46.11	52.48	84.57	87.52	96.01	98.11*	**98.45**	96.71	97.98
↓ OS	0.81	**0.00**	**0.00**	**0.00**	1.58	**0.00**	**0.00**	1.71	1.78
↓ US	4.18	1.94	1.70	**0.00**	1.20	**0.00**	**0.00**	**0.00**	0.23*
↓ ME	44.82	41.55	9.50	6.70	0.78	0.49*	**0.37**	0.68	0.78
↓ NE	45.29	40.97	10.22	6.90	0.89	0.46*	0.46*	0.48	0.68
↓ O	14.52	20.74	7.00	7.49	2.72	1.16	0.93	**0.72**	0.78*
↓ C	16.77	22.10	5.34	6.16	2.29	1.56	1.04*	**0.7**	1.53
↑ CA	65.42	67.01	86.21	87.08	93.95	97.01	**97.67**	95.86	97.24*
↑ CO	76.19	77.86	92.02	92.61	96.73	98.43*	**98.78**	96.91	98.32
↑ CC	80.30	78.34	92.68	93.26	97.02	98.46*	**98.81**	97.38	98.4
↓ I.	23.81	22.14	7.98	7.39	3.27	1.57*	**1.22**	3.09	1.68
↓ II.	4.82	4.40	1.70	1.49	0.68	0.33	0.25*	0.41	**0.20**
↑ EA	75.40	76.21	91.72		96.68	98.40*	**98.77**	97.01	98.32
↑ MS	64.29	66.79	88.03		95.10	97.65*	**98.17**	95.37	97.49
↓ RM	6.43	4.47	2.08	1.38	0.86	0.28*	**0.24**	0.61	0.30
↑ CI	76.69	77.05	92.02	92.81	96.77	98.42*	**98.78**	97.08	98.34
↓ GCE	25.79	23.94	11.76	11.76	5.55	2.84	2.33	**2.13**	2.29*
↓ LCE	20.68	19.69	8.61	8.61	3.75	2.23	1.68	**1.46**	1.61*
↓ dD	20.35	17.86	7.50		3.06	1.57	**1.21**	1.45	1.32*
↓ dM	13.25	10.62	4.69		1.96	0.99	**0.74**	0.77*	**0.74**
↓ dVI	14.51	14.22	13.99		13.80	13.71	**13.68**	**13.68**	13.70*

Color experiments - Results (2/2)





Conclusion

Take home message:

- Empirical wavelets drastically improve texture characterization,
- Convolutional Neural Networks do NOT properly characterize textures!
- Application in STM Microscopy image analysis (using a different classifier).

Future work:

- Can we design neural networks where empirical wavelets replace the convolutional layers?
- New empirical wavelets: empirical Gabor $\rightarrow$ currently in development and the first tests show improvements!
- These approaches can be used in 1D to segment oscillatory patterns in signals (e.g. EEG analysis).

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QUESTIONS?